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Best-Range Altitude for Jet-Propelled Aircraft

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Nomenclature

$C_{L\mathrm{opt}}$ = lift coefficient for $E_{\max}$
c = specific fuel consumption
$E_{\max}$ = maximum aerodynamic efficiency for parabolic polar
h = altitude, m
S = wing area
T = thrust
t = dimensionless thrust, $(TE_{\max}/W)$
V = airspeed
V_R = reference or minimum drag airspeed, $(2W/\rho SC_{L\mathrm{opt}})^{1/2}$
v = dimensionless airspeed, V/V_R
W = aircraft weight
W_F = fuel weight
X = range
X_R = reference range, $\{8V_{Ri0}E_{\max}[1-(1-\zeta)^{1/2}]\}/(c_{11}\sigma_{11}^{0.5})$
y = dimensionless exponent for specific fuel consumption
z = dimensionless exponent for thrust
ζ = fuel-to-initial-weight ratio, W_F/W_i
ρ = air density
σ = relative air density with respect to sea level, ρ/ρ_0

Subscripts

a = flight condition for v equal to $3^{1/4}$
br = best (maximum) range condition
c = absolute ceiling condition (based on initial weight)
i = initial weight condition
m = maximum thrust condition
0 = sea level, 0 m
11 = tropopause level, 11,000 m

Introduction

In classical flight mechanics,[1–3] the range of a jet-propelled aircraft is calculated using simplified assumptions about aerodynamic characteristics, e.g., parabolic polar, no compressibility effects; engines, e.g., thrust and specific fuel consumption are proportional to powers of air density; and flight profiles, e.g., quasisteady and level flight. For a given altitude, the maximum range with a constant altitude–constant lift coefficient flight program is achieved with dimensionless airspeed constant and equal to $3^{1/4}$. When increasing altitude to maximize this maximum range, two problems appear: First, exponents fitting thrust and specific fuel consumption as powers of the air densities are different in the troposphere and stratosphere and different models must be used in those layers. Second, the available thrust has a finite maximum value and is impossible to fly at $v = 3^{1/4}$ above certain altitude. This Note analytically solves these problems, offers a closed formulation for the best range and best-range altitude, and compares them with the ceiling range and ceiling altitude.

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Basic Equations

The usual approaches for thrust and specific fuel consumption of a jet engine are

$$t/t_{11} = (\sigma/\sigma_{11})^z, \qquad c/c_{11} = (\sigma/\sigma_{11})^y \tag{1}$$

where t_{11} depends on thrust setting and c_{11} is a constant.

The quasisteady level flight condition for an aircraft with a parabolic polar gives a relationship between t and v,

$$t = \tfrac{1}{2}[v^2 + (1/v^2)] \tag{2}$$

The well-known range equation for a jet-propelled aircraft with constant altitude–constant lift coefficient flight program is[1–3]

$$X = \frac{4V_{Ri0}E_{\max}(1-\sqrt{1-\zeta})\sigma_{11}^y}{c_{11}}\frac{1}{\sigma^{y+0.5}}\frac{v^3}{v^4+1} \tag{3}$$

and the classical unconstrained best-range condition for a fixed altitude (or fixed σ) is

$$v = 3^{\frac{1}{4}} = 1.316, \qquad t = 2/3^{\frac{1}{2}} = 1.155$$

Thus,

$$X_a = \frac{3^{\frac{3}{4}}V_{Ri0}E_{\max}(1-\sqrt{1-\zeta})\sigma_{11}^y}{c_{11}}\frac{1}{\sigma^{y+0.5}}$$

The preceding expression increases as σ decreases (that is, as altitude increases), and the maximum value for best range would be reached at the absolute ceiling. At the absolute ceiling, however, the only possible airspeed is $v = 1$ [and consequently, by means of Eq. (2), $t = 1$]. Therefore, it is necessary to change the formulation to analyze the dimensionless interval $1 \le v \le 1.316$, with the constraint that in any point of the trajectory the thrust must be lesser or equal to the maximum available thrust ($T \le T_m$).

Additional problems arise due to the different exponents z and y used in the troposphere and stratosphere. Typical values for a turbojet are[1] troposphere: $z = 0.7$ and $y = 0.2$ and stratosphere: $z = 1.0$ and $y = 0$. Using these values, two separate formulations for the troposphere and stratosphere are proposed.

Troposphere ($z = 0.7$, $y = 0.2$)

By the changing of the two degrees of freedom, from v and σ to v and t_{11}, Eq. (3) is transformed to

$$X = \frac{8V_{Ri0}E_{\max}(1-\sqrt{1-\zeta})}{c_{11}\sigma_{11}^{0.5}}t_{11}\frac{v^5}{(v^4+1)^2}$$

By the use of a reference range defined as

$$X_R = \frac{8V_{Ri0}E_{\max}(1-\sqrt{1-\zeta})}{c_{11}\sigma_{11}^{0.5}}$$

a condensed expression for the range is obtained

$$\frac{X}{X_R} = t_{11}\frac{v^5}{(v^4+1)^2} \tag{4}$$

where $t_{11} = T_{11}E_{\max}/W$ is maintained constant in the flight.

For the best-range condition, Eq. (4) has a maximum for

$$t_{11\mathrm{br}} = t_{11mi} = (T_{11m}/W_i)E_{\max}, \qquad v_{\mathrm{br}} = \left(\tfrac{5}{3}\right)^{\frac{1}{4}} = 1.136$$

Therefore, $t_{\mathrm{br}} = 4/15^{1/2} = 1.033$, $\sigma_{\mathrm{br}}/\sigma_{11} = (1.033/t_{11mi})^{1/0.7}$, and $X_{\mathrm{br}}/X_R = 0.2663 \times t_{11mi}$.

For the absolute ceiling condition (based on initial weight),

$$t_{11c} = t_{11mi}, \qquad v_c = 1, \qquad t_c = 1$$

$$\frac{\sigma_c}{\sigma_{11}} = \left(\frac{1}{t_{11mi}}\right)^{1/0.7}, \qquad \frac{X_c}{X_R} = 0.2500 \times t_{11mi}$$

For the $v = 3^{1/4}$ condition,

$$t_{11a} = t_{11mi}, \qquad v_a = 1.316, \qquad t_a = 1.155$$

$$\frac{\sigma_a}{\sigma_{11}} = \left(\frac{1.155}{t_{11mi}}\right)^{1/0.7}, \qquad \frac{X_a}{X_R} = 0.2468 \times t_{11mi}$$

Finally, to transform densities into altitudes (in meters), the International Standard Atmosphere (ISA) is used.[4] For $0 \le h \le 11{,}000$ m,

$$h = \frac{1}{22.558 \times 10^{-6}}\left[1 - \left(\frac{\sigma/\sigma_{11}}{3.3663}\right)^{1/4.256}\right]$$

The difference between ceiling altitude and best-range altitude (expressed in meters) will be

$$h_c - h_{\text{br}} = 486 - 0.01096 h_c = \frac{365}{t_{11mi}^{0.3357}}$$

Stratosphere ($z = 1.0, y = 0$)

Repeating the same steps used in the tropospheric approach,

$$X = \frac{4\sqrt{2}V_{Ri0}E_{\max}(1 - \sqrt{1-\zeta})}{c_{11}\sigma_{11}^{0.5}} t_{11}^{0.5} \frac{v^4}{(v^4+1)^{\frac{3}{2}}}$$

$$\frac{X}{X_R} = \frac{1}{\sqrt{2}} t_{11}^{0.5} \frac{v^4}{(v^4+1)^{\frac{3}{2}}} \tag{5}$$

For the best-range condition,

$$t_{11\text{br}} = t_{11mi} = (T_{11m}/W_i)E_{\max}, \qquad v_{\text{br}} = 2^{\frac{1}{4}} = 1.189$$

Therefore, $t_{\text{br}} = 3/8^{1/2} = 1.061$, $\sigma_{\text{br}}/\sigma_{11} = 1.061/t_{11mi}$, and $X_{\text{br}}/X_R = 0.2722 \times (t_{11mi})^{0.5}$.

For the absolute ceiling condition,

$$t_{11c} = t_{11mi}, \qquad v_c = 1, \qquad t_c = 1$$

$$\frac{\sigma_c}{\sigma_{11}} = \frac{1}{t_{11mi}}, \qquad \frac{X_c}{X_R} = 0.2500 \times (t_{11mi})^{0.5}$$

For the $v = 3^{1/4}$ condition,

$$t_{11a} = t_{11mi}, \qquad v_a = 1.316, \qquad t_a = 1.155$$

$$\frac{\sigma_a}{\sigma_{11}} = \frac{1.155}{t_{11mi}}, \qquad \frac{X_a}{X_R} = 0.2652 \times (t_{11mi})^{0.5}$$

By means of the ISA,[4] for $11{,}000 < h \le 20{,}000$ m,

$$h = 11{,}000 - \frac{\ln(\sigma/\sigma_{11})}{157.69 \times 10^{-6}}$$

and the difference between ceiling altitude and best-range altitude (expressed in meters) now will be

$$h_c - h_{\text{br}} = 375$$

Different Cases Depending on t_{11mi}

Five different cases appear, which depend on the t_{11mi} value.

1) For case 1, $t_{11mi} \le 1$: the ceiling is in the troposphere (or tropopause) and best range and $v = 3^{1/4}$ are in the troposphere.

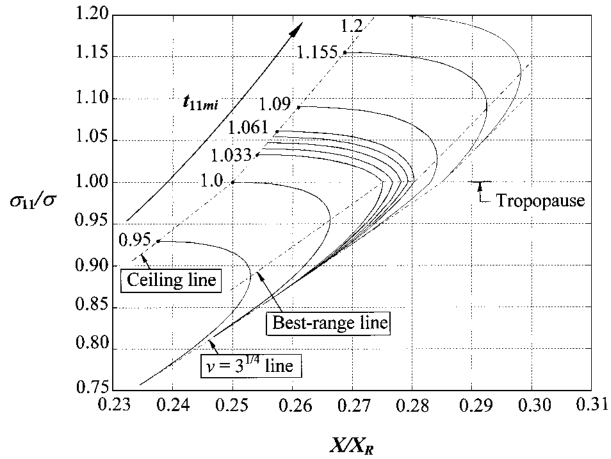

Fig. 1 Dimensionless range vs σ_{11}/σ and t_{11mi}.

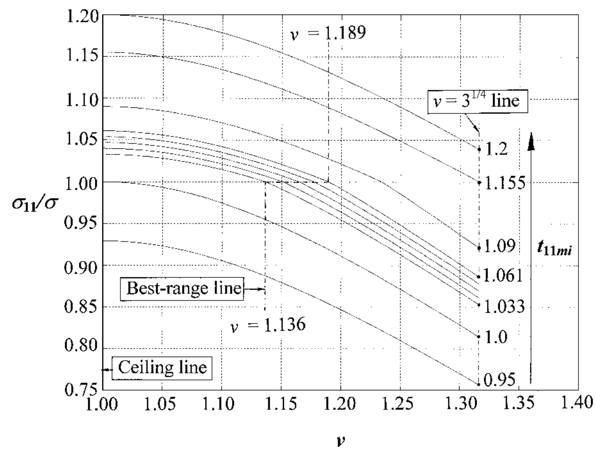

Fig. 2 Dimensionless airspeed vs σ_{11}/σ and t_{11mi}.

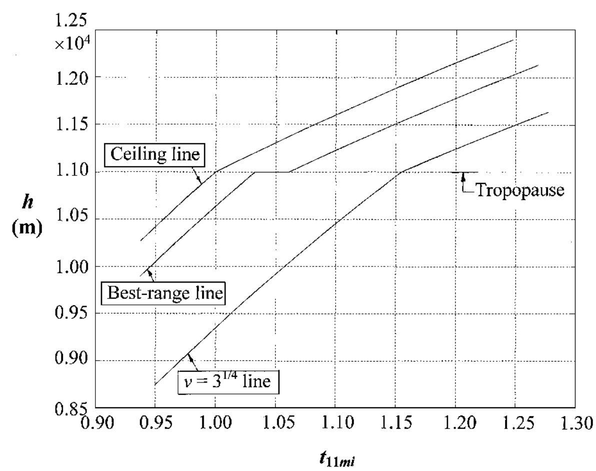

Fig. 3 Altitudes for ceiling, best range, and $v = 3^{1/4}$ vs t_{11mi}.

2) For case 2, $1 < t_{11mi} < 1.033$: the ceiling is in the stratosphere and best range and $v = 3^{1/4}$ are in the troposphere.

3) For case 3, $1.033 \le t_{11mi} \le 1.061$: the ceiling is in the stratosphere, best range is in the tropopause, and $v = 3^{1/4}$ is in the troposphere. For this particular case,

$$t_{11\text{br}} = t_{\text{br}} = t_{11mi}, \qquad v_{\text{br}} = \sqrt{t_{11mi} + \sqrt{t_{11mi}^2 - 1}}, \qquad \sigma_{\text{br}} = \sigma_{11}$$

$$\frac{X_{\text{br}}}{X_R} = t_{11mi}\frac{v_{\text{br}}^5}{\left(v_{\text{br}}^4+1\right)^2} = \frac{1}{\sqrt{2}} t_{11mi}^{0.5} \frac{v_{\text{br}}^4}{\left(v_{\text{br}}^4+1\right)^{\frac{3}{2}}}$$

4) For case 4, $1.061 < t_{11mi} \le 1.155$: the ceiling and best range are in the stratosphere, and $v = 3^{1/4}$ is in the troposphere (or tropopause).

5) For case 5, $1.155 < t_{11mi}$: the ceiling, best range, and $v = 3^{1/4}$ are in the stratosphere.

For cases 1–5, Fig. 1 shows X/X_R as a function of σ_{11}/σ and t_{11mi}, and Fig. 2 gives v as a function of σ_{11}/σ and t_{11mi} (note that σ_{11}/σ increases as h increases). In Figs.1 and 2, lines for ceiling, best range, and $v = 3^{1/4}$ are represented. Finally, in Fig. 3, altitudes for ceiling, best range, and $v = 3^{1/4}$ are shown as functions of t_{11mi}.

Conclusions

The best-range altitude for a jet-propelled aircraft with a constant altitude–constant lift coefficient flight program is neither the absolute ceiling altitude nor the altitude in which $v = 3^{1/4}$ for maximum thrust setting. The dimensionless maximum thrust in the tropopause based on the initial weight (t_{11mi}) appears as the unique and universal parameter to determine the exact values of ceiling range, best range, ceiling altitude, and best-range altitude. Because of the different models applied in the troposphere and stratosphere for thrust and specific fuel consumption, the best range is placed as follows: in the troposphere when $t_{11mi} < 1.033$, in the stratosphere when $t_{11mi} > 1.061$, and in the tropopause when $1.033 \leq t_{11mi} \leq 1.061$. Finally, the difference between the absolute ceiling altitude and the best-range altitude is always 375 m, if the ceiling and the best range are in the stratosphere ($t_{11mi} > 1.061$), and the difference depends on t_{11mi} for the other cases, but maintains the same order of magnitude, e.g., for $h_c = 11{,}000$ m, $h_c - h_{\mathrm{br}} = 365$ m and for $h_c = 8000$ m, $h_c - h_{\mathrm{br}} = 398$ m.

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Errata

Improvement to Numerical Predictions of Aerodynamic Flows Using Experimental Data Assimilation

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THE authors' affiliation should have been cited as UMIST, Manchester, England M60 1QD, United Kingdom. Also, D. Drikakis's footnote should have read: Lecturer, Mechanical Engineering Department, P.O. Box 88; currently Reader (Associate Professor), Queen Mary and Westfield College, Engineering Department, University of London, London, England E9 4NS, United Kingdom. Senior Member AIAA. AIAA regrets these errors.